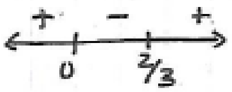
AP Calculus BC Review

1) $f(x) = 3x^3 - 3x^2 + 5$

$f'(x) = 9x^2 - 6x = 0$

$3x(3x-2) = 0$

$x = 0 \quad x = \frac{2}{3}$



$f(x)$ is inc on $(-\infty, 0)$ & $(\frac{2}{3}, \infty)$ b/c $f'(x) > 0$

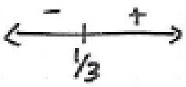
$f(x)$ is dec on $(0, \frac{2}{3})$ b/c $f'(x) < 0$

$f(x)$ has a local max @ $x = 0$ b/c $f'(x)$ changes from + to −.

$f(x)$ has a local min @ $x = \frac{2}{3}$ b/c $f'(x)$ changes from − to +.

$f''(x) = 18x - 6 = 0$

$x = \frac{1}{3}$



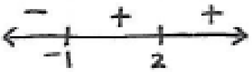
$f(x)$ is concave down on $(-\infty, \frac{1}{3})$ to $f''(x) < 0$

$f(x)$ is concave up on $(\frac{1}{3}, \infty)$ to $f''(x) > 0$

$f(x)$ has a point of inflection @ $x = \frac{1}{3}$ b/c $f''(x)$ changes signs.

2) $f'(x) = 6(x+1)(x-2)^2$

$x = -1 \quad x = 2$



a) No local max

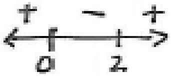
b) $f(x)$ has a local min @ $x = -1$ b/c $f'(x)$ changes from − to +.

$f''(x) = 6(x+1) \cdot 2(x-2) + 6(x-2)^2$

$= 6(x-2)[2(x+1) + x - 2]$

$= 6(x-2)(3x)$

$x = 2 \quad x = 0$



c) $f(x)$ has a point of inflection @ $x = 0, 2$ b/c $f''(x)$ changes signs.

3) $f(x) = e^x + \sin x$

$(0, 1) \quad f'(x) = e^x + \cos x$

$f'(0) = 2$

$y - 1 = 2(x - 0)$

$y = 1 + 2(x - 0)$

$\boxed{L(x) = 2x + 1}$

4) $y' < 0 \rightarrow y$ is dec

$y'' < 0 \rightarrow y$ is concave down

$\boxed{\text{point } T}$

5) $s(t) = 3 + 4t - 3t^2 - t^3$

$v(t) = 4 - 6t - 3t^2$

$a(t) = -6 - 6t$

$-6 - 6t = 0$

$t = -1$

$-3t^2 - 6t + 4 = 0$

$3t^2 + 6t - 4 = 0$

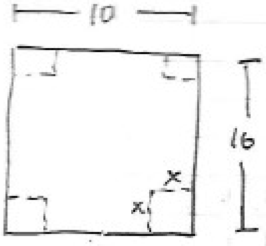
$t = \frac{-6 \pm \sqrt{36 - 4(3)(-4)}}{6}$

$= \frac{-6 \pm \sqrt{84}}{6}$

$= \frac{-6 + \sqrt{84}}{6}$

≈ 0.527

6)



$V = x(10-2x)(16-2x)$

$V = 160x - 52x^2 + 4x^3$

$V' = 160 - 104x + 12x^2$

$V' = 40 - 26x + 3x^2$

$3x^2 - 26x + 40 = 0$

$(3x-20)(x-2) = 0$

$x = \cancel{20/3}$ $\quad x = 2$

Not Possible

$V''(x) = 6x - 26$

$V''(2) < 0$

when 2 in of cardboard are cut from the corners, the max volume of the box will be attained.

7) $A = \pi r^2$ $\quad \frac{dr}{dt} = -\frac{2}{\pi}$ $\quad r = 10$ m

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\quad \frac{dA}{dt} = 2\pi(10)(-\frac{2}{\pi})$

$= -40$ m²/sec

8) $y = \sin^2 x - 3x$

$y' = 2\sin x \cos x - 3$

$-1 \le \sin x \le 1$

$-1 \le \cos x \le 1$ $\quad$ Therefore $2\sin x \cos x < 3$

y is decreasing

9) a) f has a max @ $x = -2$ b/c f' Δs signs from + to −.

b) f has a min @ $x = 0$ b/c f' Δs signs from − to +.

c) f is concave up on $(-1, 1)$ & $(2, 3]$ b/ f' is increasing.